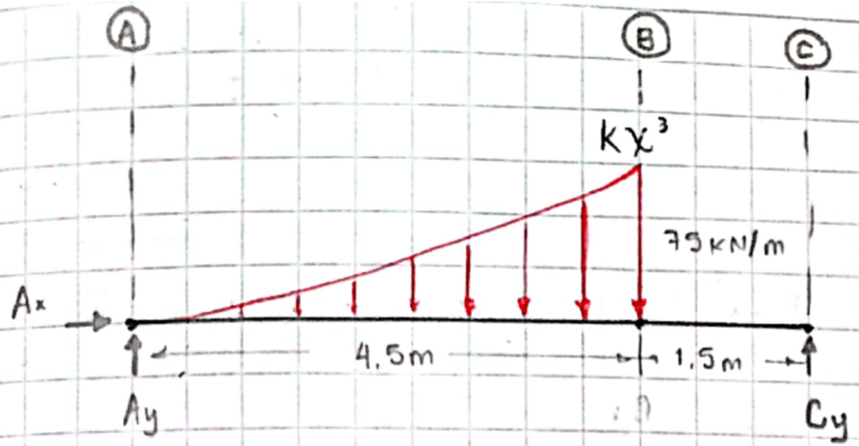


- Cálculo de K

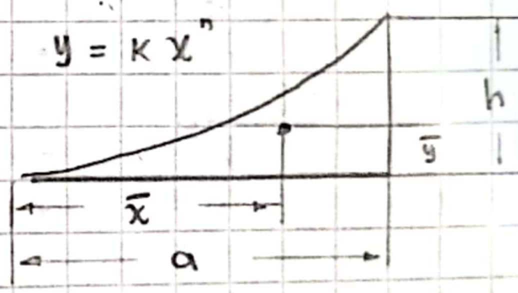
$$Q = K\chi^3 \quad ; \quad \text{si} \quad \chi = 4{,}5\text{m} \quad ; \quad Q = 75\ \text{kN/m}$$

$$K = \frac{Q}{\chi^3} \quad \text{sustituyendo;} \quad K = \frac{75\ \text{kN/m}}{(4{,}5\text{m})^3} = 0{,}82\ \frac{\text{kN}}{\text{m}^4}$$

- Cálculo de área y centroide

Con base en el libro de Mecánica Vectorial para Ingenieros, 9 ed, pag. 225: Enjuta general.

$$\bar{x} = \frac{n+1}{n+2} \cdot a \quad ; \quad \bar{y} = \frac{n+1}{4n+2} \cdot h$$



$$A = \frac{ah}{n+1}$$

Sustituyendo; ($\bar{x}$ y A)

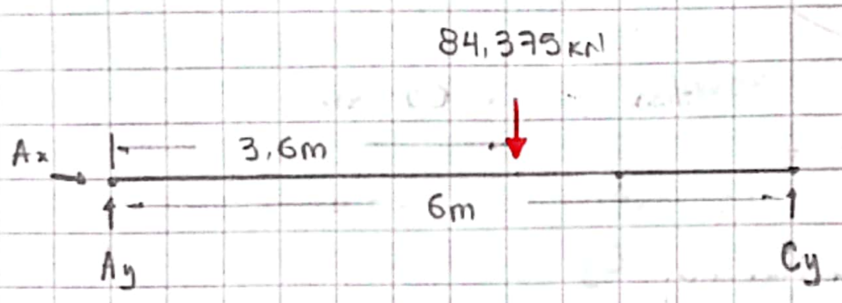
$$\bar{x} = \frac{(3)+1}{(3)+2} \cdot (\chi) = \frac{4}{5}\chi = 0{,}8\ X$$

$$A = \frac{(\chi)(K\chi^3)}{(3)+1} = \frac{K\chi^4}{4} = 0{,}21\ \chi^4$$

Cuando $X = 4{,}5\text{m}$

$$\bar{x} = 0{,}8 \cdot 4{,}5\text{m} = 3{,}6\text{m} \quad ; \quad A = \frac{0{,}82 \cdot (4{,}5\text{m})^4}{4} = 84{,}375\ \text{kN}$$

Por lo tanto, el sistema se puede reemplazar:



• $\Sigma F_x = 0$; $A_x = 0\,kN$ (No hay fuerza axial)

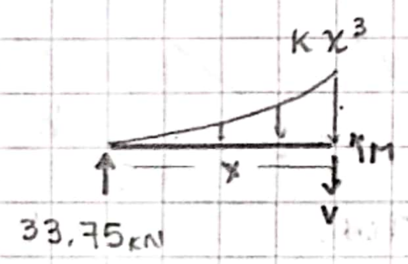
• $\Sigma M_A = 0$; $-(3.6m)(84,375\,kN) + (6m)\cdot B_y = 0$

$$C_y = \frac{(3,6m)(84,375\,kN)}{(6m)} = 50,625\,kN \uparrow$$

• $\Sigma F_y = 0$; $A_y + 50,625\,kN - 84,375\,kN = 0$

$$A_y = 84,375\,kN - 50,625\,kN = 33,75\,kN \uparrow$$

● Corte AB: $0 \leq x \leq 4.5m$



• $\Sigma V = 0$;

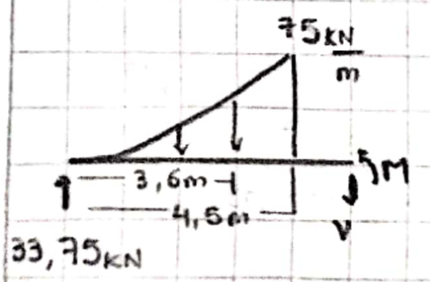
$$-33,75 + 0,21\,x^4 + V = 0$$

$$V = 33,75 - 0,21\,x^4$$

• $\Sigma M = 0$; $M + (0,21\,x^4)(x - 0,8\,x) - 33,75\,x = 0$

$$M = 33,75\,x - 0,041\,x^5$$

● Corte BC $4.5m \leq x \leq 6m$



• $\Sigma V = 0$; $-33,75 + 84,375$

$$V = -55,625$$

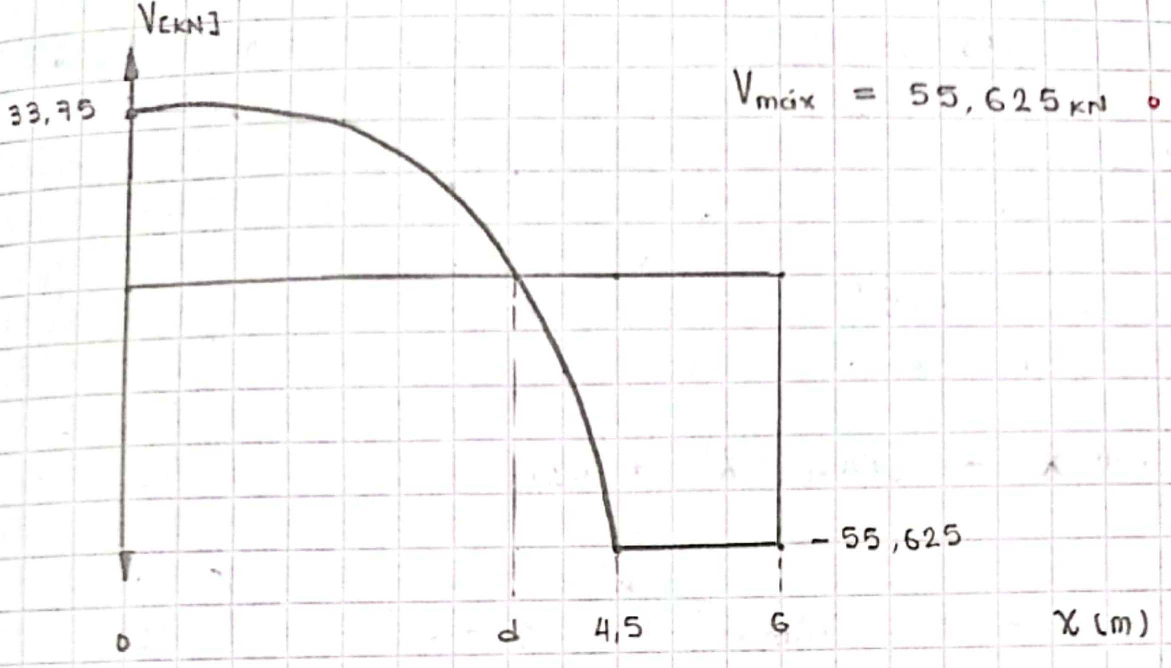
• $\Sigma M = 0$; $-33,75\,x + 84,375\,(x - 3,6)$

$$M = -50,625\,x + 303,75$$

Escaneado con CamScanner

Ⓐ Diagrama de cortante y momento

• Diagrama de cortante:

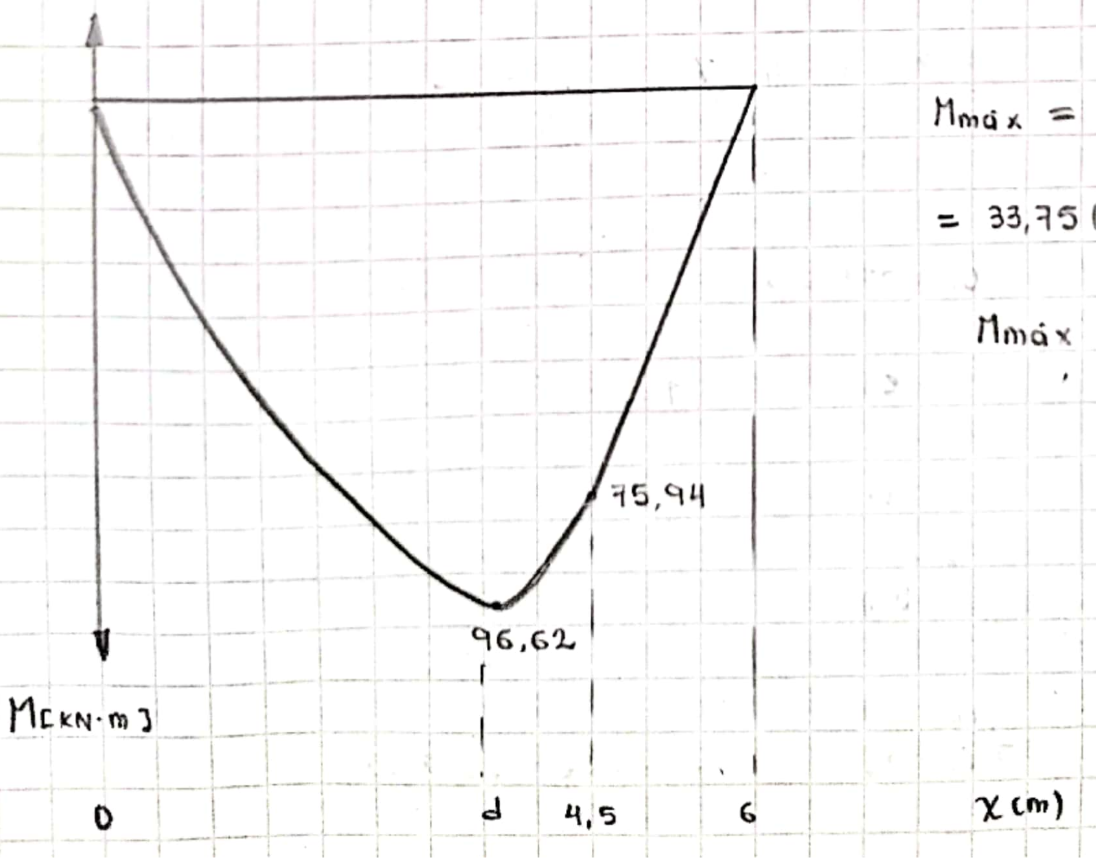


$V_{máx} = 55,625_{KN}$

$$V = 33,75 - 0,21x^4 = 0;$$

$$d = x = \left(\frac{33,75}{0,21}\right)^{1/4} = 3,58m$$

• Diagrama de momento:



$M_{máx} = M(d)$

$= 33,75(3,58) - 0,041(3,58)^5$

$M_{máx} = 96,62\ KN \cdot m$

- Ecuación de deflexión:

Se sabe que $\dfrac{d^2y}{dx^2} = \dfrac{M}{EI}$

→ Tramo AB:

$$EI\,\frac{d^2y}{dx^2} = 33{,}75\,X - 0{,}041\,X^5$$

$$EI\,\frac{dy}{dx} = 16{,}875\,X^2 - 0{,}006859\,X^6 + C_1 \qquad (1)$$

$$EI\,y = 5{,}625\,X^3 - 0{,}0009798\,X^7 + C_1 X + C_2 \qquad (2)$$

→ Tramo CB:

$$EI\,\frac{d^2y}{dx^2} = -50{,}625\,X + 303{,}75$$

$$EI\,\frac{dy}{dx} = -25{,}3125\,X^2 + 303{,}75\,X + C_3 \qquad (3)$$

$$EI\,y = -8{,}4375\,X^3 + 151{,}875\,X^2 + C_3 X + C_4 \qquad (4)$$

- Consideraciones geométricas

$$y_{(0)_{AB}} = 0 \qquad (5)$$

$$y_{(6)_{BC}} = 0 \qquad (6)$$

$$y_{(4,5)_{AB}} = y_{(4,5)_{BC}} \qquad (7)$$

$$\frac{dy}{dx}_{(4,5)_{AB}} = \frac{dy}{dx}_{(4,5)_{BC}} \qquad (8)$$

(2) en (5)

$$5{,}625\cdot(0)^3 - 0{,}0009798(0)^7 + C_1\cdot(0) + C_2 = 0$$

$$C_2 = 0$$

(4) en (6)

$$-8{,}4375\cdot(6)^3 + 151{,}875\cdot(6)^2 + C_3\cdot(6) + C_4 = 0$$

$$3645 + 6C_3 + C_4 = 0$$

$$C_4 = -3645 - 6C_3 \qquad (9)$$

(2) y (4) en (7)

$$5{,}625\cdot(4{,}5)^3 - 0{,}0009798\cdot(4{,}5)^7 + C_1\cdot(4{,}5) = -8{,}4375(4{,}5)^3 + 151{,}875(4{,}5)^2$$

$$+ C_3\cdot(4{,}5) + C_4$$

$$475{,}96 + 4{,}5\,C_1 = 2306{,}60 + 4{,}5\,C_3 + C_4 \qquad (10)$$

(1) y (3) en (8)

$$-25{,}3125\cdot(4{,}5)^2 + 303{,}75\cdot(4{,}5) + C_3 = 16{,}875(4{,}5)^2 - 0{,}006859(4{,}5)^6 + C_1$$

$$854{,}3 + C_3 = 284{,}76 + C_1$$

$$C_1 = C_3 + 569{,}53 \qquad (11)$$

(9) y (11) en (10)

$$475{,}96 + 4{,}5\cdot(C_3 + 569{,}53) = 2306{,}6 + 4{,}5C_3 + (-3645 - 6C_3)$$

$$6C_3 = 2306{,}6 - 3645 - 2562{,}89 - 475{,}96$$

$$C_3 = \frac{-4377{,}254}{6}$$

Así; $C_3 = -729{,}5424$

en (11); $C_1 = (-729{,}5424) + 569{,}53$

$C_1 = -160{,}0112$

en (9); $C_4 = -3645 - 6\cdot(-729{,}5424)$

$C_4 = 732{,}2545$

Finalmente;

→ Tramo AB: $0 \leq x \leq 4{,}5m$

$$EI\frac{dy}{dx} = 16{,}875X^2 - 0{,}006859X^6 - 160{,}0112$$

$$EI\,y = 5{,}625X^3 - 0{,}0009798X^7 - 160{,}0112X$$

→ Tramo BC $4{,}5m \leq X \leq 6m$

$$EI\frac{dy}{dx} = -25{,}3125X^2 + 303{,}75X - 729{,}5424$$

$$EI\,y = -8{,}4375X^3 + 151{,}875X^2 - 729{,}5424X + 732{,}2545$$

- Deflexión máxima:

$$\frac{dy}{dx} = 0 \quad ; \quad 16{,}875x^2 - 0{,}006859x^6 - 160{,}0112 = 0$$

Al solucionar la ecuación; $X = 3{,}142m$

$$EI\,y_{máx} = 5{,}625\cdot(3{,}142m)^3 - 0{,}0009798\cdot(3{,}142m)^7 - 160{,}0112\cdot(3{,}142m)$$

$$EI\,y_{máx} = -331{,}2389\ KN\cdot m^3$$

Material : Acero

Con base en el libro de Mecánica de Materiales. de Beer, 5 ed, pag. 747, de las propiedades típicas del acero:

$$\sigma_{UT} = \sigma_{UC} = \tau_U = 400 \text{ MPa}$$ B

$$E = 200 \text{ GPa}$$

Se determinó que $M_{máx} = 96{,}62 \text{ kN}\cdot\text{m}$

Se conoce qué $\sigma = \dfrac{M \cdot c}{I} = \dfrac{M}{S_x}$

Despejando; $S_x = \dfrac{M_{máx}}{\sigma_U} = \dfrac{96{,}62 \text{ kN}\cdot\text{m}}{400 \times 10^3 \text{ kPa}}$

$$S_x = 0{,}00024 \text{ m}^3 \cdot \left(\frac{1000 \text{ mm}}{1 \text{ m}}\right)^3 = 241\,563{,}48 \text{ mm}^3$$

$$S_x \geq 241{,}56 \times 10^3 \text{ mm}^3$$

Con base en el mismo libro, pag. 753, se observa que el perfil óptimo es:

C W200 × 26,6

$A = 3\,390 \text{ mm}^2$

$S_x = 249 \times 10^3 \text{ mm}^3$

$I = 25{,}8 \times 10^6 \text{ mm}^4$

$t_w = 5{,}8 \text{ mm}$

$d = 207 \text{ mm}$

Verificando esfuerzo cortante

$$\tau_{máx} = \frac{V_{máx}}{A_{Alma}} = \frac{V_{máx}}{d \cdot t_w} = \frac{55{,}625 \text{ kN}}{(5{,}8 \text{ mm})(207 \text{ mm})} \times \left(\frac{1000 \text{ mm}}{1 \text{ m}}\right)^2$$

$$= 46{,}33 \text{ MPa}$$ ✓ Cumple porque es menor que 400 MPa

Con base en la sección supuesta;

$$E = 200 \text{ GPa}$$
$$I = 25{,}8 \times 10^6 \text{ mm}^4$$

Así;
$$EI = 200 \times 10^6 \frac{\text{kN}}{\text{m}^2} \cdot 25{,}8 \times 10^6 \text{mm}^4 \cdot \left(\frac{1 \text{ m}}{1000 \text{mm}}\right)^4$$

$$= 5160 \text{ kN} \cdot \text{m}^2$$

Deflexión máxima:

$$y_{máx} = \frac{-331{,}2389 \text{ kN} \cdot \text{m}^3}{5160 \text{kN} \cdot \text{m}^2} = -0{,}064 \text{ m} \cdot \frac{1000 \text{mm}}{1 \text{m}}$$

$$y_{máx} = -64{,}19 \text{ mm}$$

- Curva de deflexión:

A Complemento