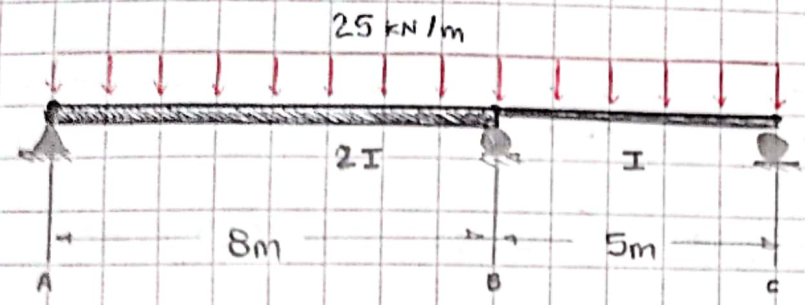
Taller Giro deflexión - Vigas

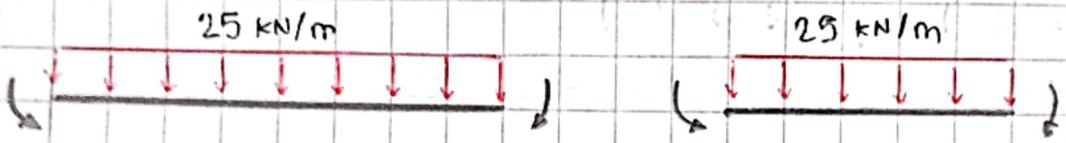
$E = 175\ GPa$

$I = 500 \times 10^{6}\ mm^4$

$= 500 \times 10^{-6}\ m^4$



Momentos de empotramiento



$$M_{AB}^{F} = \frac{(25\ kN/m)(8m)^2}{12} = 133{,}33\ kN \cdot m = -M_{BA}^{F}$$

$$M_{BC}^{F} = \frac{(25\ kN/m)(5m)^2}{12} = 52{,}083\ kN \cdot m = -M_{CB}^{F}$$

Aplicación de ecuaciones de giro y deflexión

$$K_{AB} = \frac{2 \times 500 \times 10^{-6}\ m^4}{8m} = 125 \times 10^{-6}$$

$$K_{BC} = \frac{500 \times 10^{-6}\ m^4}{5m} = 100 \times 10^{-6}$$

$$M_{AB} = M_{AB}^{F} + EK_{AB}(4\theta_{AB} + 2\theta_{BA} + 6\Delta/L)$$

$$M_{BA} = M_{BA}^{F} + EK_{AB}(2\theta_{AB} + 4\theta_{BA} + 6\Delta/L)$$

$$M_{BC} = M_{BC}^{F} + EK_{BC}(4\theta_{BC} + 2\theta_{CB} + 6\Delta/L)$$

$$M_{CB} = M_{CB}^{F} + EK_{BC}(2\theta_{BC} + 4\theta_{CB} + 6\Delta/L)$$

(a) Condiciones

$\theta_{BA} = \theta_{BC}$ $\quad M_{BA} + M_{BC} = 0$

$\Delta = 0$ $\quad M_{AB} = 0$

$M_{CB} = 0$

$$M_{AB}^F + 4\theta_{AB} E K_{AB} + 2\theta_{BA} \cdot E K_{AB} = 0 \quad (1)$$

$$M_{BA}^F + 2\theta_{AB} E K_{AB} + 4\theta_{BA} E K_{AB} + M_{BC}^F + 4\theta_{BC} E K_{BC} + 2\theta_{CB} \cdot E K_{BC} = 0$$

$$M_{BA}^F + 2\theta_{AB} \cdot E K_{AB} + 4E\theta_{BA}(K_{AB} + K_{BC}) + 2\theta_{CB} \cdot E \cdot K_{BC} + M_{BC}^F = 0 \quad (2)$$

$$M_{CB}^F + 2\theta_{BC} E K_{BC} + 4\theta_{CB} \cdot E K_{CB} = 0 \quad (3)$$

De (1);

$$2\theta_{AB} \cdot E \cdot K_{AB} = \frac{-2\theta_{BA} \cdot E \cdot K_{AB} - M_{AB}^F}{2}$$

$$= -\theta_{BA} \cdot E \cdot K_{AB} - \frac{M_{AB}^F}{2} \quad (4)$$

De (3);

$$2\theta_{CB} \cdot E \cdot K_{BC} = \frac{-2\theta_{BC} \cdot E \cdot K_{BC} - M_{CB}^F}{2}$$

$$= -\theta_{BC} \cdot E \cdot K_{BC} - \frac{M_{CB}^F}{2} \quad (5)$$

(4) y (5) en (2)

$$M_{BA}^F - \theta_{BA} \cdot E \cdot K_{AB} - \frac{M_{AB}^F}{2} + 4E\theta_{BA}(K_{AB} + K_{BC}) + M_{BC}^F$$

$$- \theta_{BA} \cdot E \cdot K_{BC} - \frac{M_{CB}^F}{2} = 0$$

$$\theta_{BA} \cdot E \left[-K_{AB} + 4(K_{AB} + K_{BC}) - K_{BC} \right] = -M_{BA}^F + \frac{M_{AB}^F + M_{CB}^F}{2} - M_{BC}^F$$

$$\theta_{BA} \cdot E = \frac{-2M_{BA}^F + M_{AB}^F + M_{CB}^F - 2M_{BC}^F}{6(K_{AB} + K_{BC})}$$

$$\theta_{BA} = \frac{-2M_{BA}^F + M_{AB}^F + M_{CB}^F - 2M_{BC}^F}{6E(K_{AB} + K_{BC})}$$

$$\theta_{BA} = 0,001032$$

De (4); $$\theta_{AB} = -\frac{\theta_{BA}}{2} - \frac{M_{AB}^F}{4 \cdot E \cdot K_{AB}}$$

$$\theta_{AB} = -0,00204$$

De (5); $$\theta_{CB} = -\frac{\theta_{BA}}{2} - \frac{M_{CB}^F}{4 \cdot E \cdot K_{BC}}$$

$$\theta_{CB} = 0,0002282$$

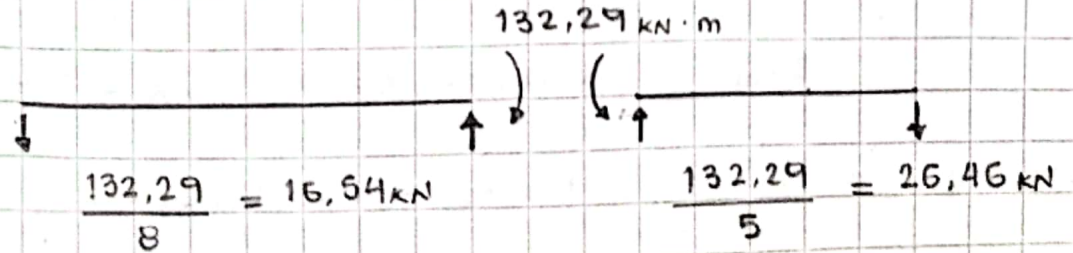
Así; $M_{AB} = 0$

$M_{BA} = -132,29 \text{ kN} \cdot \text{m}$

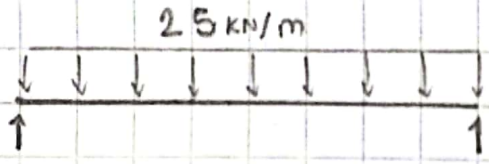
$M_{BC} = 132,29 \text{ kN} \cdot \text{m}$

$M_{CB} = 0$

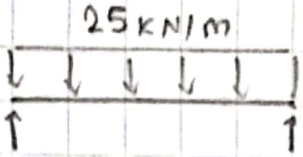
- Equilibrando momentos



- Equilibrando cargas

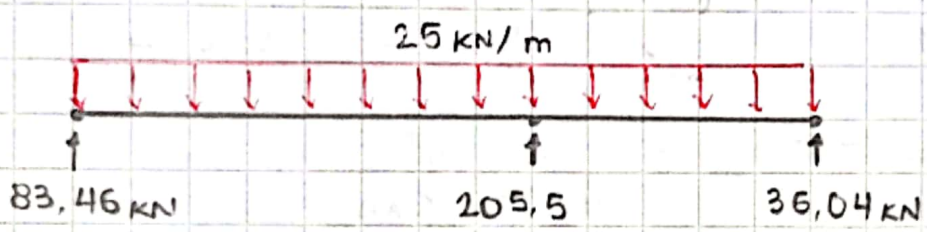


$\frac{(25)(8)}{2} = 100$

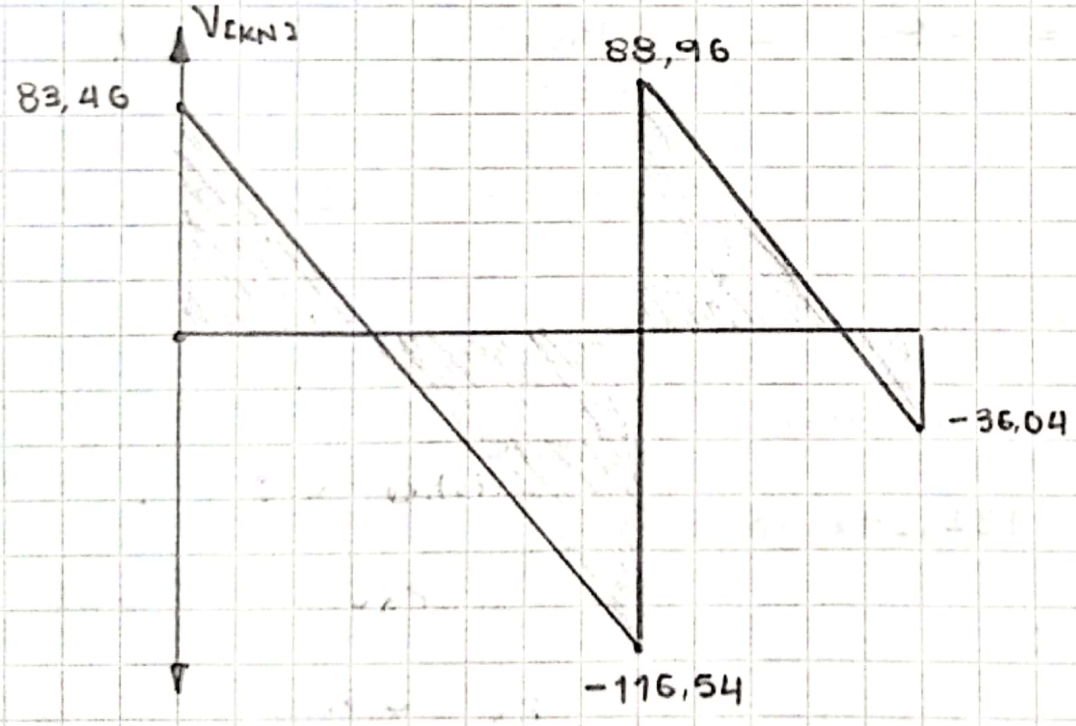


$\frac{(25)(5)}{2} = 62,5$

- Superponiendo;



- Diagrama de cortante:



- $M_{máx_1} = 83,46 \cdot X - 12,5\,X^2$

$83,46 - 25X = 0$; $X = 3,34\,m$

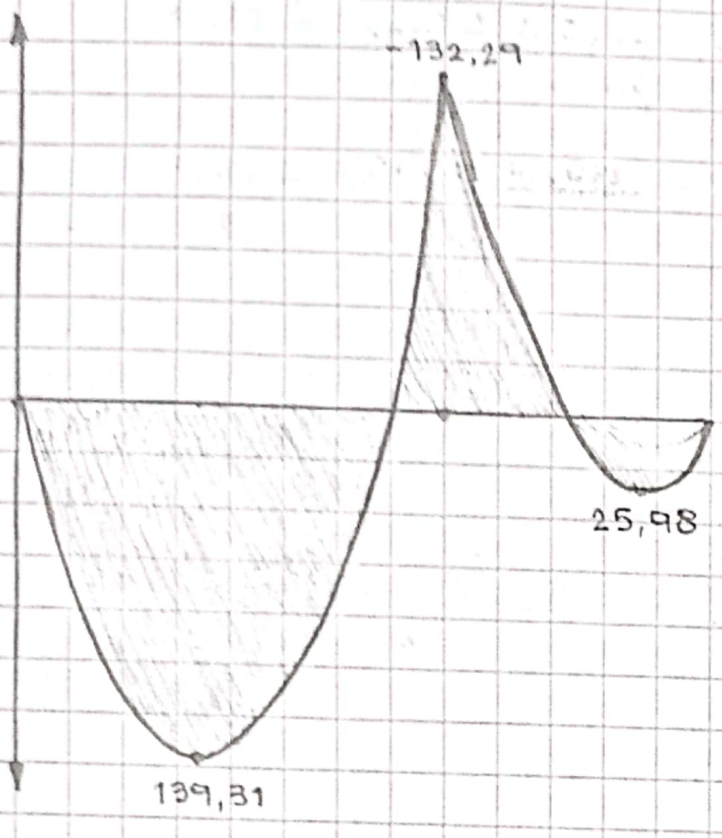
$M_{máx_1} = 139,31\,kN \cdot m$

- $M_{máx_2} = 36,04X - 12,5\,X^2$

$36,04 - 25X = 0$; $X = 1,44\,m$

$M_{máx_2} = 10,06\,kN \cdot m$

- Diagrama de momento:



M[KN·m]

(b) Condiciones;

$\theta_{BA} = \theta_{BC}$ $\quad M_{BA} + M_{BC} = 0$

$\Delta = 0.005m$ $\quad M_{AB} = 0$

$M_{CB} = 0$

$$\rightarrow M_{AB}^{F} + 4\theta_{AB} E K_{AB} + 2\theta_{BA} E K_{AB} + 0,75\Delta E K_{AB} = 0 \quad (1)$$

$$\rightarrow M_{BA}^{F} + M_{BC}^{F} + 2\theta_{AB} E K_{AB} + 4\theta_{BA} E K_{AB} + 0,75\Delta E K_{AB} + 4\theta_{BC} E K_{BC} + 2\theta_{CB} E K_{BC} + 1,2\Delta E K_{BC} = 0$$

$$M_{BA}^{F} + M_{BC}^{F} + 2\theta_{AB} E K_{AB} + 4\theta_{BC} E (K_{AB} + K_{BC}) + 2\theta_{CB} E K_{BC} + \Delta E (0,75K_{AB} + 1,2K_{BC}) = 0 \quad (2)$$

$$\rightarrow M_{CB}^{F} + 2\theta_{BC} E K_{BC} + 4\theta_{CB} E K_{CB} + 1,2\Delta E K_{BC} = 0 \quad (3)$$

$$2\theta_{AB} \cdot E \cdot K_{AB} = \frac{-2\theta_{BA} \cdot E \cdot K_{AB} - 0,75 \cdot \Delta \cdot E \cdot K_{AB} - M_{AB}^F}{2}$$

$$= -\theta_{BA} \cdot E \cdot K_{AB} - 0,375\,\Delta \cdot E \cdot K_{AB} - \frac{M_{AB}^F}{2}$$

De (3);

$$2\theta_{CB}\, E \cdot K_{BC} = \frac{-2\theta_{BA} \cdot E \cdot K_{BC} - 1,2\,\Delta E \cdot K_{BC} - M_{CB}^F}{2}$$

$$= -\theta_{BA} \cdot E \cdot K_{BC} - 0,6\,\Delta E \cdot K_{BC} - \frac{M_{CB}^F}{2}$$

(4) y (5) en (2)

$$M_{BA}^F + M_{BC}^F - EK_{AB}(\theta_{BA} + 0,375\,\Delta) - \frac{M_{AB}^F}{2} + 4\theta_{BC} E (K_{AB} + K_{BC})$$

$$- EK_{BC}(\theta_{BA} + 0,6\,\Delta) - \frac{M_{CB}^F}{2} + \Delta E (0,75\,K_{AB} + 1,2\,K_{BC}) = 0$$

Sustituyendo y resolviendo;

$\theta_{BA} = 0,0002401$

$\theta_{AB} = -0,002581$

$\theta_{CB} = -0,000876$

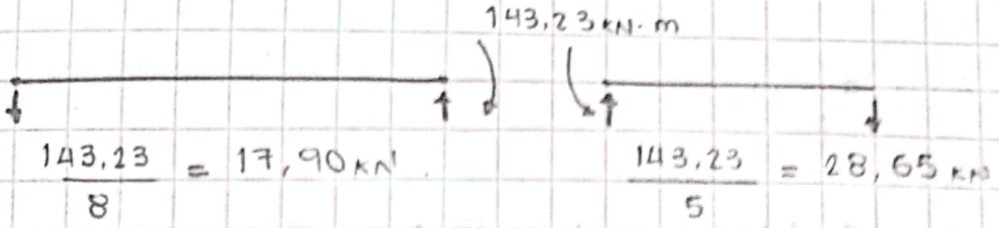
Así; $M_{AB} = 0$

$M_{BA} = -143,23$ KN·m

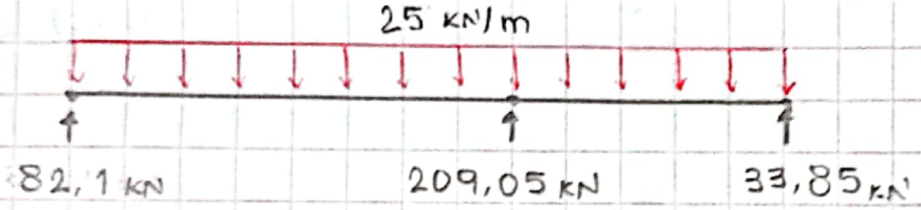
$M_{BC} = 143,23$ KN·m

$M_{CB} = 0$

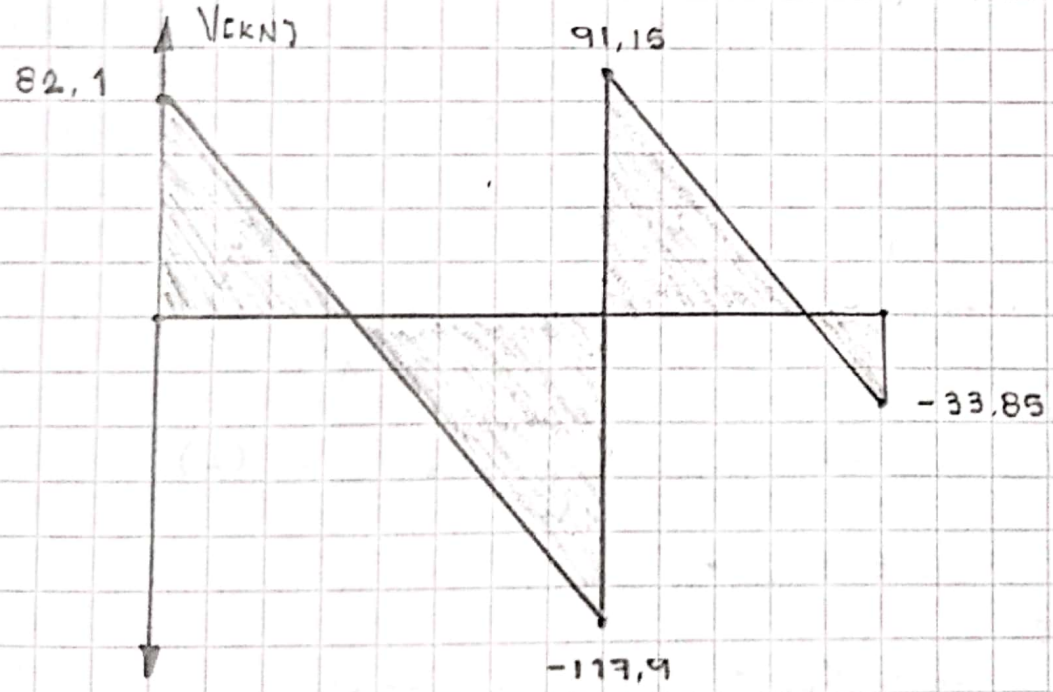
- Equilibrio de momentos:



- Equilibrio de cargas: (No varia)
- Superponiendo:



- Diagrama de cortante



$M_{máx_1} = 82,1x - 12,5x^2$

$82,1 - 25x = 0; \quad x = 3,284m$

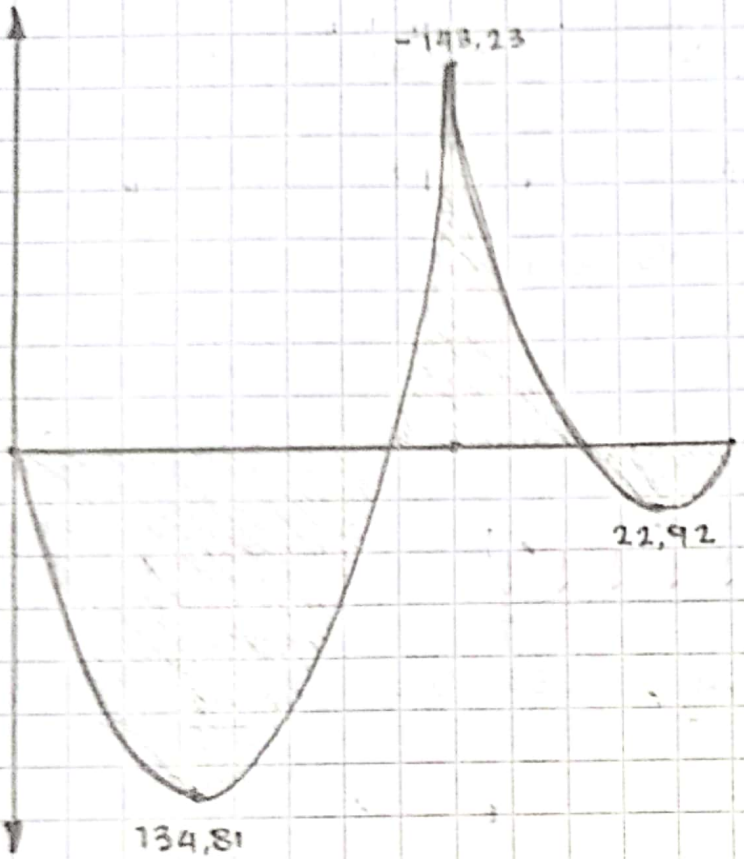
$M_{máx_1} = 134,81 KN \cdot m$

$M_{máx_2} = 33,85x - 12,5x^2$

$33,85 - 25x = 0; \quad x = 1,354m$

$M_{máx_2} = 22,9165 KN \cdot m$

● Diagrama de momento



M[KN·m]

ⓒ Condiciones;

$\theta_{AB} = 0$ $\quad M_{BA} + M_{BC} = 0$

$\theta_{BA} = \theta_{BC}$ $\quad M_{CB} = 0$

$\Delta = 0,005m$

$M_{AB} = M_{AB}^{F} + 2\theta_{BA} \cdot E \cdot K_{AB} + 0,75\Delta \cdot E \cdot K_{AB}$ (1)

$M_{BA}^{F} + 4\theta_{BA} E K_{AB} + 0,75\Delta \cdot E K_{AB} +$

$M_{BC}^{F} + 4\theta_{BA} \cdot E \cdot K_{BC} + 2\theta_{CB} E \cdot K_{BC} + 1,2\Delta \cdot E K_{BC} = 0$

$M_{BA}^{F} + M_{BC}^{F} + 4\theta_{BA} \cdot E \cdot (K_{AB} + K_{BC}) + 2\theta_{CB} \cdot E \cdot K_{BC} + \Delta E(0,75K_{AB} + 1,2K_{BC}) = 0$ (

$M_{CB}^{F} + 2\theta_{BA} \cdot E K_{BC} + 4\theta_{CB} \cdot E K_{BC} + 1,2\Delta \cdot E \cdot K_{BC} = 0$ (3)

De (3);

$$2\theta_{CB} \cdot E \cdot K_{BC} = -\frac{2\theta_{BA} \cdot E \cdot K_{BC} - 1.2\Delta \cdot E \cdot K_{BC} - M_{CB}^F}{2}$$

$$= -\theta_{BA} \cdot E \, K_{BC} - 0.6 \cdot \Delta \cdot E \cdot K_{BC} - \frac{M_{CB}^F}{2} \quad (4)$$

(4) en (2)

$$M_{BA}^F + M_{BC}^F + 4\theta_{BA} \, E \cdot (K_{AB} + K_{BC}) - \theta_{BA} \cdot E \cdot K_{BC} - 0{,}6\Delta \cdot E \cdot K_{BC}$$

$$- \frac{M_{CB}^F}{2} + \Delta E \cdot (0{,}75 K_{AB} + 1{,}2 K_{BC}) = 0$$

$$\theta_{BA} \cdot E \cdot [4(K_{AB} + K_{BC}) - K_{BC}] = -M_{BA}^F - M_{BC}^F + \frac{M_{CB}^F}{2}$$

$$- \Delta E (-0{,}6 \cdot K_{BC} + 0{,}75 K_{AB} + 1{,}2 K_{BC})$$

$$\theta_{BA} = -0{,}000567$$

$$\theta_{CB} = -0{,}000473$$

Así;

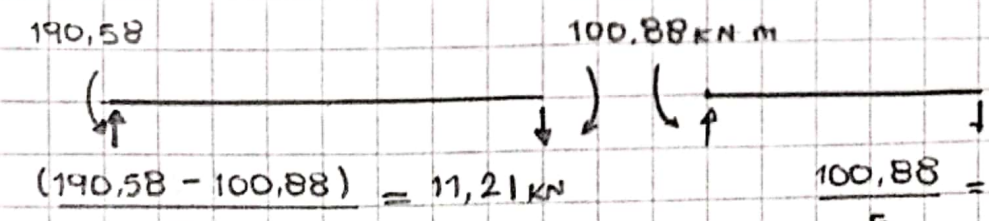
$$M_{AB} = 190{,}58 \text{ KN} \cdot \text{m}$$

$$M_{BA} = -100{,}88 \text{ KN} \cdot \text{m}$$

$$M_{BC} = 100{,}88 \text{ KN} \cdot \text{m}$$

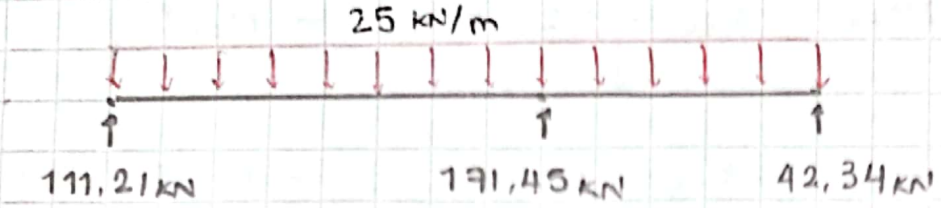
$$M_{CB} = 0 \text{ KN} \cdot \text{m}$$

- Equilibrio de momentos:

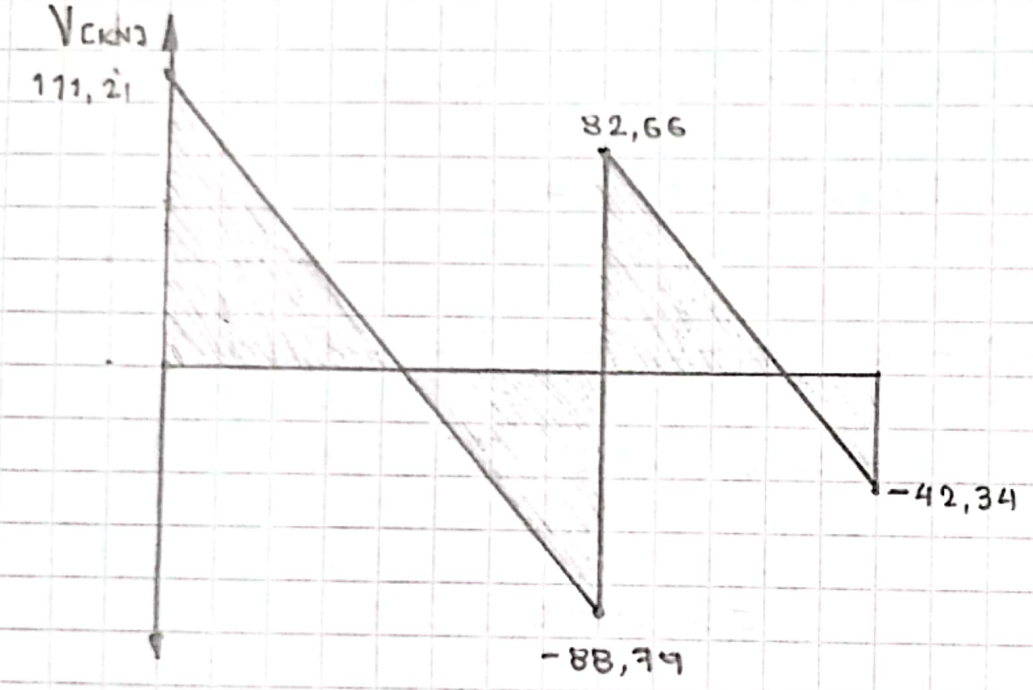


$$\frac{(190{,}58 - 100{,}88)}{8} = 11{,}21 \text{ KN} \qquad \frac{100{,}88}{5} = 20{,}16 \text{ KN}$$

- Superponiendo;



- Diagrama de cortante:



$M_{máx_1} = -190,58 + 111,21X - 12,5X^2$

$111,21 - 25X = 0$; $X = 4,4484m$

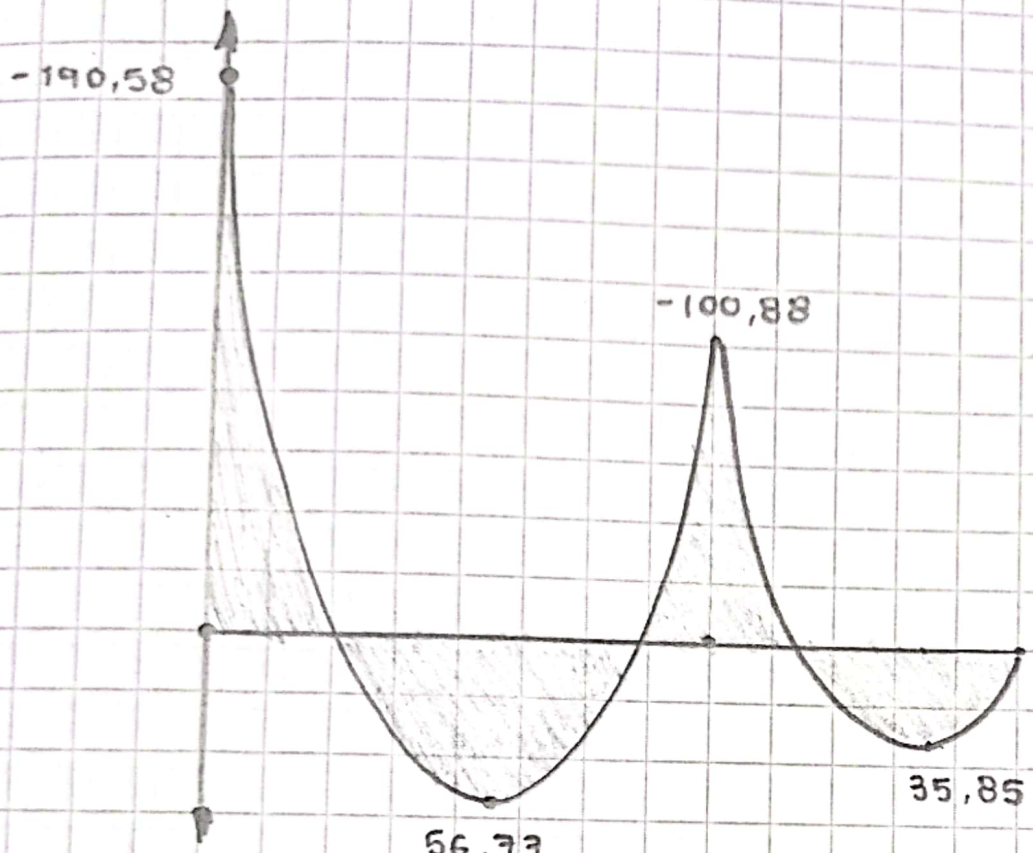
$M_{máx_1} = 56,97\ kN \cdot m$

$M_{máx_2} = 42,34X - 12,5X^2$

$42,34 - 25X = 0$; $X = 1,6936m$

$M_{máx_2} = 35,8535\ kN \cdot m$

- Diagrama de momento:



[KN·m]